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NETWORK SYNTHESIS FOR  
PRESCRIBED TRANSIENT RESPONSE  
USING TRIGONOMETRIC SERIES APPROXIMATIONS

by

WILLIAM BLOUNT RODMAN IV

Course VI

June 18, 1952

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NETWORK SYNTHESIS FOR PRESCRIBED TRANSIENT  
RESPONSE USING TRIGONOMETRIC SERIES APPROXIMATIONS

by

WILLIAM BLOUNT RODMAN IV

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B.S., U.S. Naval Academy

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SUBMITTED IN PARTIAL FULFILLMENT OF THE  
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(1952)

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June 18, 1952.

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Thesis Supervisor

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thesis  
R67

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(1961)

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(1962)

REMARKS OF SENATOR

Department of Biological Sciences

June 15, 1962

Revised by

Thesis Supervisor

Department of Biological Sciences

## ABSTRACT

### NETWORK SYNTHESIS FOR PRESCRIBED TRANSIENT RESPONSE USING TRIGONOMETRIC SERIES APPROXIMATIONS

by

WILLIAM BLOUNT RODMAN IV

SUBMITTED TO THE DEPARTMENT OF ELECTRICAL ENGINEERING  
ON 18 JUNE, 1952, IN PARTIAL FULFILLMENT OF THE REQUIREMENTS  
FOR THE DEGREE OF MASTER OF SCIENCE.

Dr. E. A. Guillemin has proposed a means of network synthesis for prescribed transient response which permits approximations to be made in the time domain rather than the frequency domain. The desired transient is obtained from an appropriate combination of auxiliary periodic functions such that the result cancels everywhere except over the period of the transient. The system function is obtained from the transforms of trigonometric series approximations to these auxiliary periodic functions. The system function thus obtained is not always realizable, but it appears that approximations can be made such that it will be realizable.

On the assumption that the system function be realizable, or that it can be made so, a synthesis procedure is developed. The transient is decomposed into two components having, respectively, even and odd symmetry about the midpoint. The system functions determined for these components satisfy the requirements for synthesis as a lossless network terminated in a resistance. Such networks are synthesized for each component. These two networks are then connected "back to back" to realize a network for the overall system function.

THESIS SUPERVISOR	E. A. Guillemin
TITLE	PROFESSOR OF ELECTRICAL ENGINEERING





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## INTRODUCTION

The problem of synthesizing a finite, lumped parameter, linear, passive network for prescribed transient response has been attacked by forming the ratio of the transforms of the output and input functions to give the system function  $h(s)$ . The system function has then been synthesized into a network with the necessary approximations being made in the frequency domain. Results obtained have varied from good to poor, and it appears that the degree of approximation obtained in the time domain cannot readily be determined from the degree of approximation made in the frequency domain.

A means of making the required approximations in the time domain, instead of the frequency domain, would permit control of the time form of the transient. One such means available is the finite trigonometric series, which can be made to approximate a given periodic time function to any desired degree of accuracy. An appropriate combination of periodic functions based on the desired transient can then be made such that the combination cancels everywhere except over the period of the transient.





## CHAPTER I

## DERIVATION OF THE TRANSFER FUNCTION TO BE REALIZED

The following method of representing a transient of finite duration by an appropriate combination of semi-periodic functions has been proposed.<sup>1</sup> Define the desired transient as  $f(t)$ .

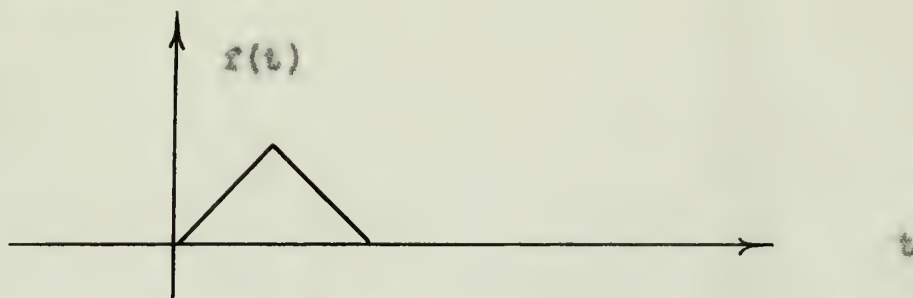


Fig. 1  
The Desired Transient

Define a semi-periodic function,  $f_p(t)$  as follows:

$$f_p(t) \equiv \begin{cases} 0 & t < 0 \\ f(t) & 0 < t < \frac{\tau}{2} \\ 0 & \frac{\tau}{2} < t < \tau \end{cases}$$

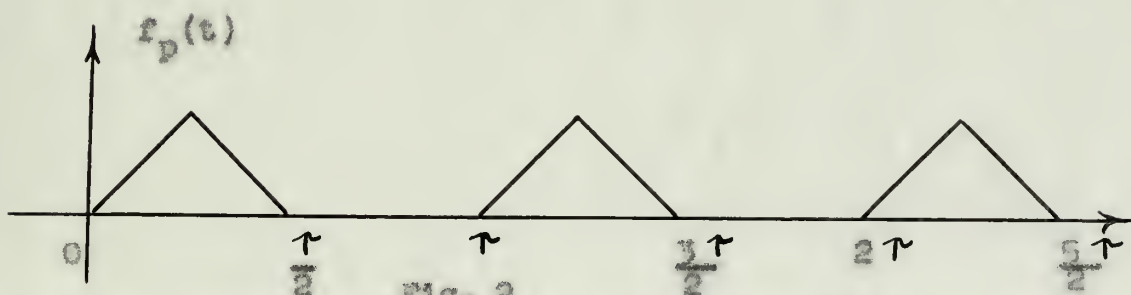
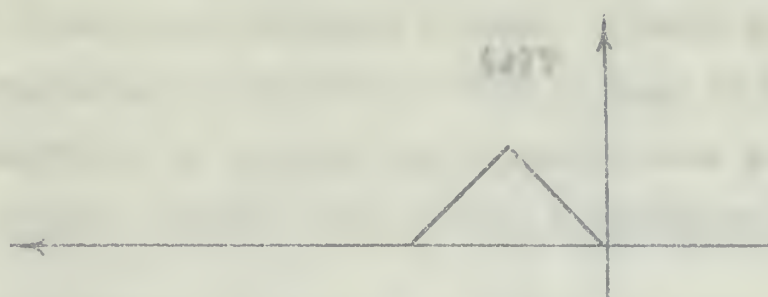


Fig. 2  
Semi-Periodic Function  $f_p(t)$  Derived from  $f(t)$

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$$\left. \begin{array}{l} 2 > 1 \\ 1 > 2 > 0 \\ 1 > 2 > 1 \end{array} \right\} \equiv 121_3$$



$f_p(t)$  is representable in a Fourier series as:

$$f_p(t) = \sum_{k=-\infty}^{\infty} \alpha_k e^{jk\omega t} \quad (t > 0)$$

and its transform  $h_p(s)$  is represented as:

$$h_p(s) = \sum_{k=-\infty}^{\infty} \frac{\alpha_k}{s - jk\omega}.$$

Now define two additional functions,  $f_1(t)$  and  $f_2(t)$  as follows:

$$f_1(t) \equiv f_p(t) + f_p(t - \frac{\tau}{2})$$

$$f_2(t) \equiv f_p(t) - f_p(t - \frac{\tau}{2})$$

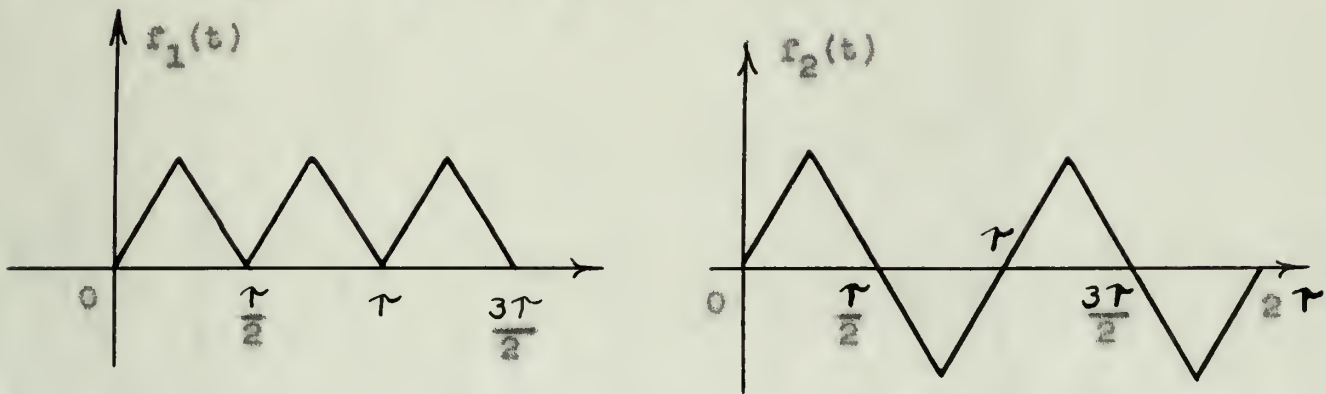


Fig. 3

Functions  $f_1(t)$  and  $f_2(t)$ , Derived From  $f_p(t)$

The function

$$f_p(t - \frac{\tau}{2}) = \sum_{k=-\infty}^{\infty} \alpha_k e^{jk\omega(t - \frac{\tau}{2})} = \sum_{k=-\infty}^{\infty} \alpha_k e^{jk\omega t} \cdot e^{-jk\omega \frac{\tau}{2}}.$$



$f_p(t)$  is periodic in  $t$  with period  $T$ :

$$f_p(t) = \sum_{k=-\infty}^{\infty} f(t - kT) \quad (t > 0)$$

and its Fourier series is expanded as

$$f_p(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \quad \omega_0 = \frac{2\pi}{T}$$

Now define two additional functions:  $f_1(t)$  and  $f_2(t)$  as

follows:

$$f_1(t) \equiv f(t) \cdot \left(1 - \frac{t}{T}\right)$$

$$f_2(t) \equiv f(t) \cdot \left(1 - \frac{t}{T}\right)$$

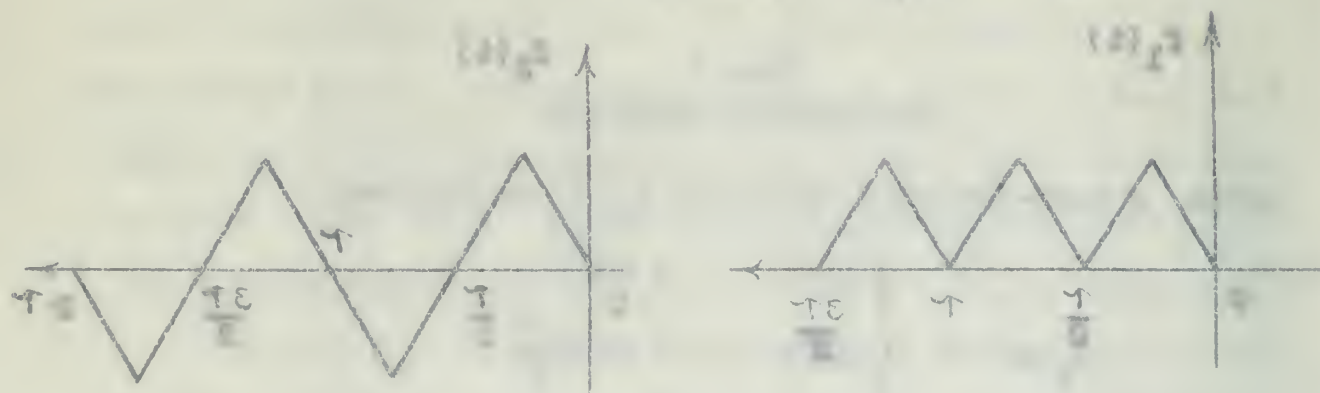


Fig. 2

Functions  $f_1(t)$  and  $f_2(t)$  derived from  $f(t)$

The function

$$f_p(t) = \sum_{k=-\infty}^{\infty} f(t - kT) = \sum_{k=-\infty}^{\infty} \left(1 - \frac{t - kT}{T}\right) f(t - kT)$$

Note that  $\frac{\omega\tau}{2} = \pi$

$$\text{then } f_p(t - \frac{\tau}{2}) = \sum_{k=-\infty}^{\infty} \alpha_k e^{jk\omega t} e^{-jk\omega\tau} = \sum_{k=0, \pm 2, 4, \dots}^{\infty} \alpha_k e^{jk\omega t} \\ - \sum_{k=\pm 1, 3, 5, \dots}^{\infty} \alpha_k e^{jk\omega t}$$

$$\text{whence } f_1(t) = 2 \sum_{k=0, \pm 2, 4, \dots}^{\infty} \alpha_k e^{jk\omega t}$$

$$\text{and } f_2(t) = 2 \sum_{k=\pm 1, 3, 5, \dots}^{\infty} \alpha_k e^{jk\omega t}$$

The transforms of  $f_1(t)$  and  $f_2(t)$  are:

$$h_1(s) = h_p(s) \left[ 1 + e^{-\frac{s\tau}{2}} \right] = 2 \sum_{k=0, \pm 2, 4, \dots}^{\infty} \frac{\alpha_k}{s - jk\omega}$$

$$h_2(s) = h_p(s) \left[ 1 - e^{-\frac{s\tau}{2}} \right] = 2 \sum_{k=\pm 1, 3, 5, \dots}^{\infty} \frac{\alpha_k}{s - jk\omega}$$

Following a line of physical reasoning, these functions are combined to form a new function,  $h(s)$  given by  $h(s) \equiv \frac{h_1 h_2}{h_1 + h_2}$ . Direct substitution shows that  $h(s) = \frac{h_p(s)}{2} [1 - e^{-s\tau}]$ . The inverse transform of  $h(s)$  is readily recognized as

$$\frac{1}{2} \{f_p(t) - f_p(t - \tau)\} = \frac{1}{2} f(t).$$

Let  $\{a_n\}$  be a sequence of real numbers. Then  $\sum_{n=1}^{\infty} a_n$  converges if and only if  $\sum_{n=1}^{\infty} |a_n|$  converges.

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The following theorem is due to Cauchy and is one of the most important results in the theory of series.

$\sum_{n=1}^{\infty} a_n$  converges if and only if  $\left\{ \sum_{k=n}^{\infty} a_k \right\}$  converges to zero.

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$\sum_{n=1}^{\infty} a_n$  converges if and only if  $\left\{ \sum_{k=n}^{\infty} a_k \right\}$  converges to zero.

In general the Fourier series representation of  $f_p(t)$  will be an infinite series. Representing finite approximations by  $*$ , then:

$$f_p^*(t) = \sum_{k=-n}^n \alpha_k e^{jk\omega t} \quad (t > 0)$$

$$h_p^*(s) = \sum_{k=-n}^n \frac{\alpha_k}{s - jk\omega} \equiv \frac{P(s)}{Q(s)}$$

$$h_1^*(s) = \sum_{k=0,2,4,\dots}^n \frac{2\alpha_k}{s - jk\omega} \equiv \frac{P_1(s)}{Q_1(s)}$$

$$h_2^*(s) = \sum_{k=\pm 1,3,5,\dots}^n \frac{2\alpha_k}{s - jk\omega} \equiv \frac{P_2(s)}{Q_2(s)}$$

$$h_1^*(s) + h_2^*(s) = 2h_p^*(s) = \frac{2P(s)}{Q(s)}$$

$$h_1^* \cdot h_2^* = \frac{P_1 P_2}{Q_1 Q_2} = \frac{P_1 P_2}{Q}$$

$$h^*(s) = \frac{P_1 P_2}{2P}$$



The convolution of two functions  $f(t)$  and  $g(t)$  is defined as  
 $(f * g)(t) = \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$   
 which is equivalent to the integral of the product of one function and the time-reversed and shifted version of the other.

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$$

$$\frac{(f * g)(t)}{t} = \frac{1}{t} \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$$

$$\frac{(f * g)(t)}{t^2} = \frac{1}{t^2} \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$$

$$\frac{(f * g)(t)}{t^3} = \frac{1}{t^3} \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$$

$$\frac{(f * g)(t)}{t^4} = \frac{1}{t^4} \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$$

$$\frac{(f * g)(t)}{t^5} = \frac{1}{t^5} \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$$

$$\frac{(f * g)(t)}{t^6} = \frac{1}{t^6} \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$$

## CHAPTER II

## DEVELOPMENT OF A SYNTHESIS PROCEDURE

The purpose here is to develop a practical means of synthesis of a network whose system function is  $h^*(s)$ , that is, a network whose response to a unit impulse function is  $f^*(t)$ .

The necessary and sufficient conditions<sup>2</sup> for the realizability of a network whose transfer impedance<sup>#</sup> is  $h^*(s)$  are:

- (a) The numerator of  $h^*(s)$  be of degree not greater than the denominator and,
- (b) The denominator have no zeros in the right half  $s$ -plane, i.e., the denominator must be a Hurwitz polynomial.

That condition (a) is fulfilled is evident from an expansion of the functions involved. The fulfillment of condition (b) is subject to discussion beyond the scope of this work. It can be demonstrated that, using the Fourier coefficients in a finite approximation of  $f_p(t)$ , certain functions yield a denominator polynomial which is Hurwitz while others do not. There appears to be a possibility of so selecting the coefficients that the denominator polynomial will be Hurwitz. In any event, this condition must

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<sup>#</sup> Note: Throughout the following development the word "admittance" may be substituted for "impedance" without invalidating the arguments.

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 1/2

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\*(d)

and the  $\beta$ -phase is the  $\beta$ -phase of the  $\beta$ -phase.

as <sup>24</sup> necessary without such direct or de vili tation

10000 (a) \*

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To comply with Revised Administrative Code 231.01, we will:

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 CCC 0887-624X/00/071111-10\$10.00

cellulose is a chain consisting of 4,4' units

Abstracts of papers presented at the 1997 Annual Meeting of the American Psychological Association, Washington, DC, August 1-5, 1997.

While above is true, there remains to be a question of

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be met before synthesis is possible.

Assume, therefore, that means have been devised to assure the Hurwitz character of the denominator polynomial, or that it is desired to synthesize a network for a function which yields one. It is advisable, in the interests of brevity, to normalize at  $\omega = 1$ , and to shift from complex coefficients to trigonometric coefficients.

Let the duration of  $f^*(t)$  be  $\pi$ , then the period of  $f_p^*(t)$  is  $2\pi$ , and  $\omega = \frac{2\pi}{T} = 1$ .

$$h_p^*(s) = \sum_{k=-n}^n \frac{\alpha_k}{s - jk} = \frac{\alpha_0}{s} + \sum_{k=1}^n \frac{(\alpha_k + \alpha_{-k})s + jk(\alpha_k - \alpha_{-k})}{s^2 + k^2}.$$

Noting that  $(\alpha_k + \alpha_{-k})$  and  $j(\alpha_k - \alpha_{-k})$  are respectively  $a_k$  and  $b_k$ , the usual trigonometric coefficients, by defining  $a_0 \equiv \frac{1}{2\pi} \int_0^{2\pi} f_p(t) dt$ ,  $h_p^*(s)$  can be written as:

$$(1) \quad h_p^*(s) = \sum_{k=0}^n \frac{a_k s + kb_k}{s^2 + k^2} = \frac{P}{Q} = \frac{P_1}{2Q_1} + \frac{P_2}{2Q_2}$$

(2)  $h^*(s) = \frac{P_1 P_2}{2P}$ , therefore the poles of  $h^*(s)$  are located at the zeros of  $P(s)$ . These zeros are  $2n$  in number and are located in the left half plane (by assumption). No other general properties of these zeros are readily apparent, and it appears that the problem of locating them would be of



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— 22 —

and awarded a certificate of fitness at its 2001-02 session.

1. The first step is to identify the problem or question that needs to be answered. This involves understanding the context and the specific information required.

Source: *Journal of the American Statistical Association*, 1977, Vol. 72, No. 359, pp. 1000-1001.

THE UNIVERSITY OF CHICAGO

For the treatment of  $\chi^2$  in the literature

$$f(x) = \frac{1}{x} = 0 \text{ lim } x \rightarrow \infty \quad (d) \quad *$$

$$\sum_{i=1}^n x_i = 1$$

... ..

$$f(x) = \frac{1}{x^2} \quad \text{for } x \in \mathbb{R} \setminus \{0\}$$

$$\frac{1}{x^2} + \frac{1}{x^2} = \frac{2}{x^2} = \frac{2}{x^2} \sum_{n=0}^{\infty} x^{2n} = (x^2)^{-1} \sum_{n=0}^{\infty} x^{2n} \quad (1)$$

$$-0.01 \ln(1 - \frac{1}{2}) = 0.01 \ln 2 = 0.01 \times 0.693 = 0.00693 \approx 0.007$$

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some degree of difficulty. This problem can be avoided, however, if lossless networks terminated in resistances are acceptable.

Recalling from Chapter I that  $h_1^*(s)$  and  $h_2^*(s)$  are respectively twice the sums of the odd order and even order terms of  $h_p^*(s)$

$$(3) \quad h_1^*(s) = 2 \sum_{k=0, 2, 4, \dots}^n \frac{a_k s + kb_k}{s^2 + k^2} = 2 \left\{ \frac{a_0}{s} + \frac{a_2 s + 2b_2}{s^2 + 4} \right. \\ \left. + \frac{a_4 s + 4b_4}{s^2 + 16} + \dots \right\} = \frac{P_1}{Q_1}.$$

$$(4) \quad h_2^*(s) = 2 \sum_{k=1, 3, 5, \dots}^n \frac{a_k s + kb_k}{s^2 + k^2} = 2 \left\{ \frac{a_1 s + b_1}{s^2 + 1} \right. \\ \left. + \frac{a_3 s + 3b_3}{s^2 + 9} + \frac{a_5 s + 5b_5}{s^2 + 25} \dots \right\} = \frac{P_2}{Q_2}.$$

Note now that if odd  $a_k$  and even  $b_k$  are zero then  $P_1$ ,  $P_2$ , and  $Q_2$  are all even functions of  $s$ , while  $Q_1$  is an odd function of  $s$ . Also if even  $a_k$  and odd  $b_k$  are zero then  $P_1$ ,  $Q_1$ , and  $Q_2$  are even in  $s$ , while  $P_2$  is odd in  $s$ . Thus the product  $P_1 Q_2$  is always even in  $s$  and the product  $P_2 Q_1$  is always odd in  $s$ .

From equation (1) it is evident that

$$(5) \quad P = \frac{1}{2} \left\{ P_1 Q_2 + P_2 Q_1 \right\},$$

some degree of difficulty. This problem can be avoided, however, if the system is designed in such a way that the necessary

assuming that the system is such that  $\hat{p}(s)$  and  $\hat{q}(s)$  are polynomials of the same order and that the order of  $\hat{p}(s)$  is  $n$

$$\hat{p}(s) = \sum_{i=0}^n a_i s^i = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 \quad (1)$$

$$\hat{q}(s) = \sum_{i=0}^m b_i s^i = b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0$$

$$\hat{p}(s) = \sum_{i=0}^n a_i s^i = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 \quad (2)$$

$$\hat{q}(s) = \sum_{i=0}^m b_i s^i = b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0$$

where now  $\hat{p}(s)$  and  $\hat{q}(s)$  are polynomials of the same order  $n$ . The system is then designed so that the output  $\hat{y}(s)$  is equal to the input  $\hat{x}(s)$  for all values of  $s$ . This is done by setting the transfer function  $\hat{y}(s)/\hat{x}(s)$  equal to 1. The system is then designed so that the output  $\hat{y}(s)$  is equal to the input  $\hat{x}(s)$  for all values of  $s$ . This is done by setting the transfer function  $\hat{y}(s)/\hat{x}(s)$  equal to 1.

The system is then designed so that the output  $\hat{y}(s)$  is equal to the input  $\hat{x}(s)$  for all values of  $s$ .

$$\hat{y}(s) = \hat{x}(s) \quad (3)$$



so that for the special conditions that odd cosine and even sine coefficients be zero, or that even cosine and odd sine coefficients be zero, the odd and even  $s$  terms in  $P(s)$  are identified with the poles and zeros of  $h_1^*(s)$  and  $h_2^*(s)$ .

The necessary and sufficient conditions<sup>3</sup> for a polynomial with real coefficients to be a Hurwitz polynomial are that the zeros of its odd and even parts lie on the  $j$  axis where they mutually separate each other. Further, the ratio of the even <sup>to the</sup> odd parts of a Hurwitz polynomial is a reactance (susceptance) function. This implies, for  $P$  Hurwitz, that the zeros of  $P_1$  and  $P_2$  lie on the  $j$  axis and that the function  $\frac{P_1 Q_2}{P_2 Q_1}$  be a reactance function, for the special conditions considered. The following manipulations are therefore suggested:

$$h^*(s) = \frac{P_1 P_2}{2P} = \frac{P_1 P_2}{P_1 Q_2 + P_2 Q_1},$$

dividing numerator and denominator by  $P_2 Q_1$  yields:

$$h^*(s) = \frac{\frac{P_1}{Q_1}}{1 + \frac{P_1 Q_2}{P_2 Q_1}}$$

Now associate  $h(s)$  with  $Z_{12}(s) = \frac{z_{12}}{1 + z_{22}}$  which is the form for the transfer impedance of a lossless network terminated in a one ohm resistance.  $z_{12}$  and  $z_{22}$  are





respectively the transfer and driving point functions for the lossless part of the network.

In making this association the following conditions are noted:

- (a)  $z_{22}$  is a reactance function.
- (b)  $z_{12}$  has its poles and zeros restricted to the  $j$  axis.
- (c) all the poles of  $z_{12}$  are contained in  $z_{22}$ .
- (d)  $z_{22}$  has poles, at the zeros of  $P_2$ , which are not contained in  $z_{12}$ .

Conditions (a), (b), and (c) are sufficient<sup>2</sup> to insure synthesis as a lossless ladder network terminated in a resistance. The poles of  $z_{22}$  which are not contained in  $z_{12}$  represent lossless parallel tuned circuits in series with the load resistor.

At this point the formal procedure of separating  $h_1^*(s)$  into two parts, each of which fulfills one set of the requirements for this type of synthesis, would lead to a general synthesis procedure. It is more enlightening, however, to first observe the time form of the transients which these conditions represent. Using Fourier series coefficients and noting that  $f_p(t) \equiv 0$  for the interval  $\pi < t < 2\pi$

respectively the transfer and driving point functions for  
the isolated part of the network.

In making this association the following con-  
ditions are needed:

- (a)  $Z_{SC}$  is a resistance function.
- (b)  $Z_{LC}$  has its poles and zeros restricted  
to the  $j$  axis.
- (c) All the poles of  $Z_{LC}$  are cancelled  
in  $Z_{SC}$ .
- (d)  $Z_{SC}$  has poles at the roots of  $P(s)$   
which are not cancelled in  $Z_{LC}$ .

Conditions (a), (b), and (c) are sufficient to insure  
existence as a lossless ladder network terminated in a  
resistance. The poles of  $Z_{SC}$  which are not cancelled in  
the resultant lossless partial band circuit in series  
with the load resistor.

At this point the formal statement of the following  
 $H^*(s)$  into two parts, each of which satisfies one set of the  
requirements for this type of synthesis, would lead to a  
general synthesis procedure. It is more satisfactory,  
however, to first express the line form of the transmission  
which shows cancellation requirement. Using partial fraction  
expansion and noting that  $L(s) \equiv 0$  for the interval  
 $\sigma < -\delta < \sigma_0$



$$a_0 \equiv \frac{1}{2\pi} \int_0^\pi f_p(t) dt$$

$$a_k \equiv \frac{1}{\pi} \int_0^\pi f_p(t) \cos kt dt$$

$$b_k \equiv \frac{1}{\pi} \int_0^\pi f_p(t) \sin kt dt$$

Since even cosine terms and odd sine terms have even symmetry about  $\frac{\pi}{2}$ , if  $f_p(\frac{\pi}{2} + t) = f_p(\frac{\pi}{2} - t)$ , then odd  $a_k$  and even  $b_k$  are zero. And since odd cosine terms and even sine terms have odd symmetry about  $\frac{\pi}{2}$ , if  $f_p(\frac{\pi}{2} + t) = -f_p(\frac{\pi}{2} - t)$ , then even  $a_k$  and odd  $b_k$  are zero. Thus the special conditions occur when the transient has even or odd symmetry about its midpoint.

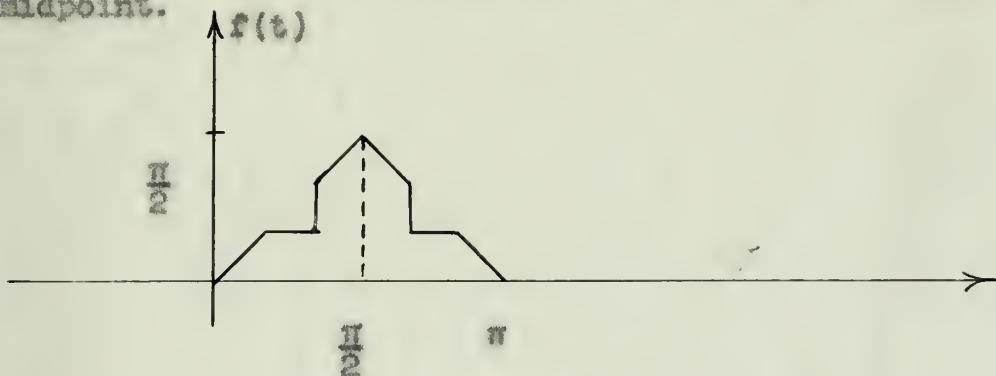


Fig. 4

Transient For Which Resulting Odd  $a_k$  And Even  $b_k$  Are Zero

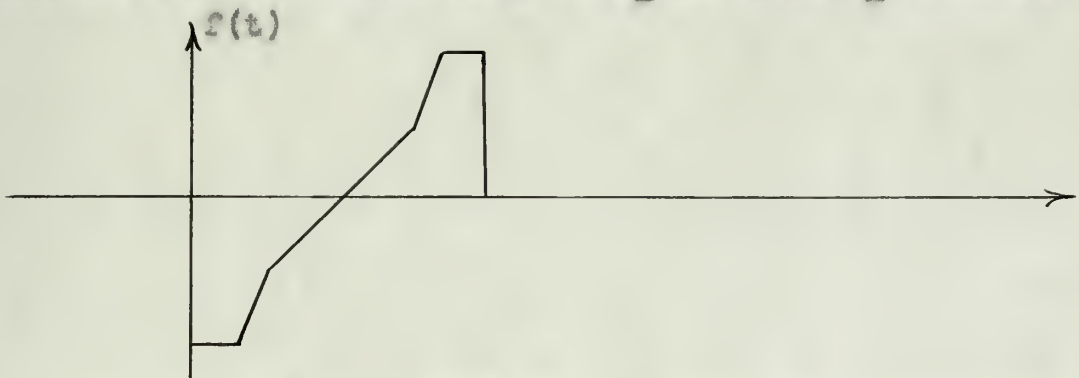


Fig. 5

Transient For Which Resulting Even  $a_k$  And Odd  $b_k$  Are Zero





Any arbitrary transient may be considered as the sum of two other transients, each having one of the above types of symmetry. To synthesize a network for an arbitrary transient response, then, first decompose the transient into its components having odd and even symmetry about the midpoint. Synthesize a network for each component. After suitable impedance leveling, connect the networks back to back and the overall transfer impedance function is realized.<sup>2</sup>

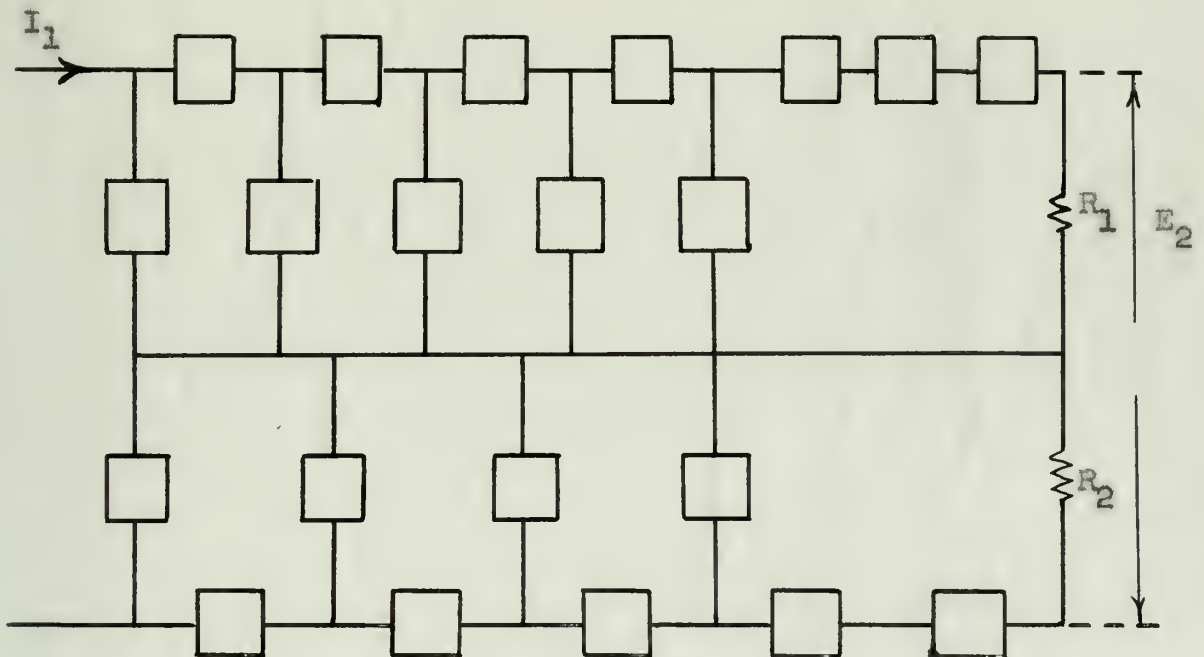


Fig. 6

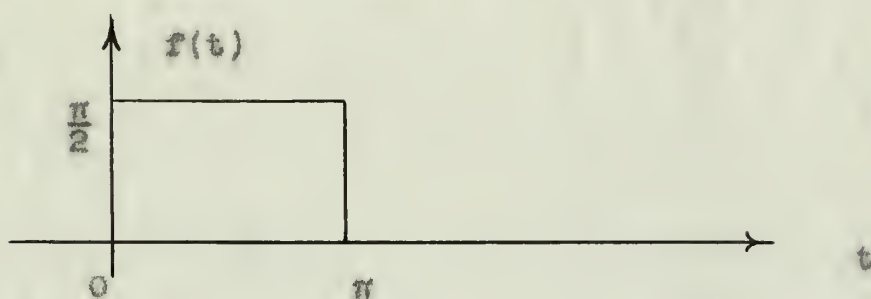
A Form Of Network To Synthesize For A Realizable Transient Response



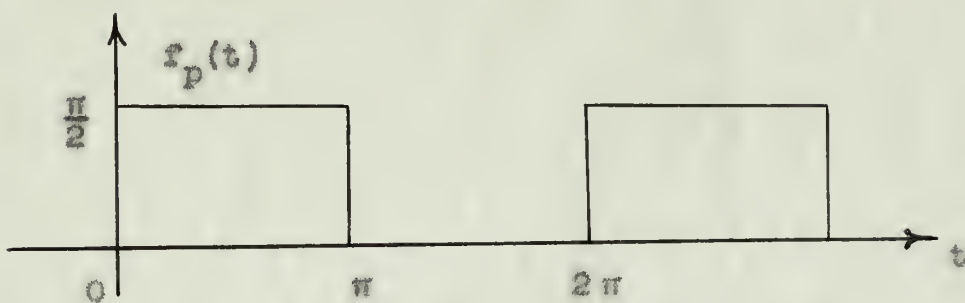
## CHAPTER III

## AN EXAMPLE OF THE SYNTHESIS PROCEDURE

Let the desired transient response to a unit impulse input be a rectangular one as shown:



Then  $f_p(t)$  is



$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f_p(t) dt = \frac{\pi}{4}$$

$$a_k = \frac{1}{\pi} \int_0^{2\pi} f_p(t) \cos kt dt = \frac{1}{\pi} \int_0^{\pi} \frac{\pi}{2} \sin kt dt = 0 \quad (k \neq 0)$$

$$b_k = \frac{1}{\pi} \int_0^{2\pi} f_p(t) \sin kt dt = \frac{1}{\pi} \int_0^{\pi} \frac{\pi}{2} \sin kt dt = \frac{1}{2k} (1 - \cos k\pi)$$

$$\therefore b_k = \frac{1}{k} \text{ for } k \text{ odd}$$

$$b_k = 0 \text{ for } k \text{ even.}$$



THE INTEGRAL

THE DEFINITION OF THE INTEGRAL

Let  $f$  be a bounded function defined on a closed interval  $[a, b]$ .

Choose  $P$  as a partition of  $[a, b]$  and let

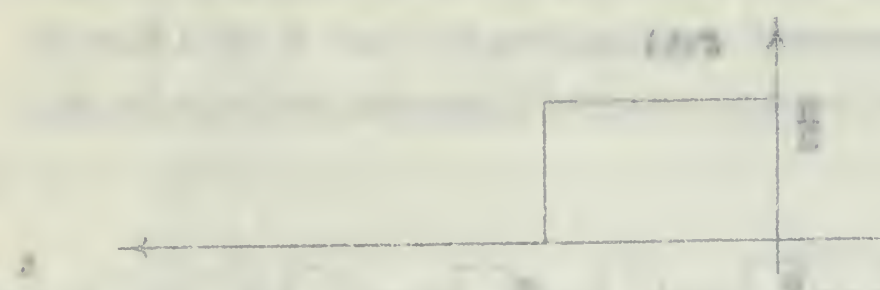


FIGURE 1



$$\frac{1}{b-a} = \int_a^b f(x) dx = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f(x_k) = \frac{1}{n} \sum_{k=1}^n \frac{1}{b-a} = \frac{1}{b-a}$$

$$(n \neq \infty) \quad \frac{1}{b-a} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{1}{b-a} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{1}{b-a} = \frac{1}{b-a}$$

$$(n \neq \infty) \quad \frac{1}{b-a} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{1}{b-a} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{1}{b-a} = \frac{1}{b-a}$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{1}{b-a} = \frac{1}{b-a} \quad \therefore$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{1}{b-a} = \frac{1}{b-a}$$

Note that odd  $a_k$  and even  $b_k$  are zero since the transient has even symmetry about its midpoint. In this instance all  $a_k$  are zero except  $a_0$ . This fact serves to simplify the network rather than invalidate the procedures developed.

Assume that it has been established that an approximation of six terms will be sufficiently accurate.

$$h_p^*(s) = \sum_{k=0}^6 \frac{a_k s + k b_k}{s^2 + k^2} = \frac{\pi/4}{s} + \frac{1}{s^2 + 1} + \frac{1}{s^2 + 9} + \frac{1}{s^2 + 25}$$

$$h_1^*(s) = \frac{\pi/4}{s} = \frac{P_1}{Q_1}$$

$$h_2^*(s) = \frac{1}{s^2 + 1} + \frac{1}{s^2 + 9} + \frac{1}{s^2 + 25} = \frac{P_2}{Q_2}$$

$$P_1 = \frac{\pi}{4}$$

$$Q_1 = s$$

$$\begin{aligned} P_2 &= (s^2 + 1)(s^2 + 9) + (s^2 + 1)(s^2 + 25) + (s^2 + 9)(s^2 + 25) \\ &= 3(s^2 + 4.61132)(s^2 + 18.7702) \end{aligned}$$

$$Q_2 = (s^2 + 1)(s^2 + 9)(s^2 + 25)$$

$$z_{12}(s) = \frac{P_1}{Q_1} = \frac{\pi/4}{s}$$

$$z_{22}(s) = \frac{P_1 Q_2}{P_2 Q_1} = \frac{(\pi/4)(s^2 + 1)(s^2 + 9)(s^2 + 25)}{3s(s^2 + 4.61132)(s^2 + 18.7702)}$$



Removing the factor of  $\frac{\pi}{12}$  results in the following:

$$z_{22} = s + \frac{2.606}{s} + \frac{4.686s}{s^2 + 4.61132} + \frac{4.094s}{s^2 + 18.72202}$$

$$R = \frac{12}{\pi}$$

$$z_{12} = \frac{3}{s}$$

If only the rectangular form of the transient be required the impedance level may be reduced by a factor of  $\frac{3}{2.606}$  yielding

$$z_{22} = s + \frac{2.606}{s} + \frac{4.686s}{s^2 + 4.61132} + \frac{4.094s}{s^2 + 18.72202}$$

$$R = 3.318$$

$$z_{12} = \frac{2.606}{s}$$

The network is then realized as

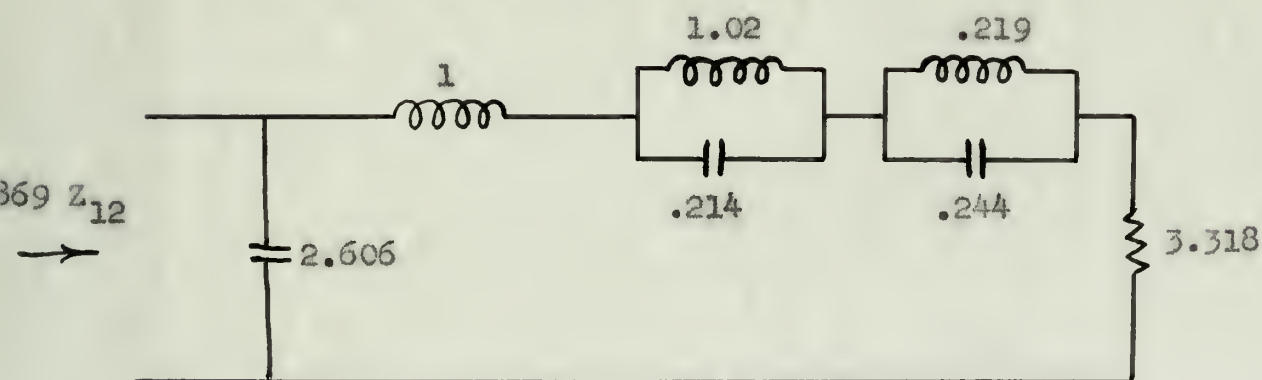


Fig. 8

Network For Approximating Rectangular Transient Response To  
A Unit Impulse

(Ohms, Henries, Farads)



Knowing the factor of  $\frac{1}{s}$  results in the following:

$$E_{22} = \frac{1.000}{s} + \frac{1.000}{s + 1.000} + \frac{1.000}{s + 1.000}$$

$$E_{22} = \frac{1}{s}$$

$$E_{22} = \frac{1}{s}$$

If only the rectangular form of the transform be required the impedance level can be reduced by a factor of

$$\frac{1}{2.000}$$

$$E_{22} = \frac{1.000}{s} + \frac{1.000}{s + 1.000} + \frac{1.000}{s + 1.000}$$

$$E_{22} = \frac{1}{s}$$

$$E_{22} = \frac{1.000}{s}$$

The network is then realized as

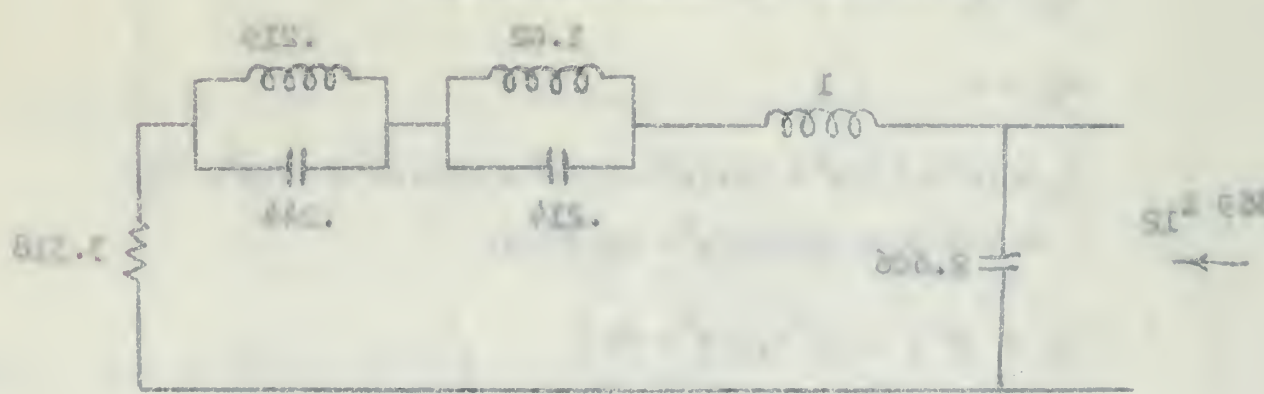


Fig. 2

Network for Approximating exponential function response to a unit impulse (Open, transfer, 1/s)

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1. The first step is to identify the problem or question that needs to be answered. This involves understanding the context and the specific information required.

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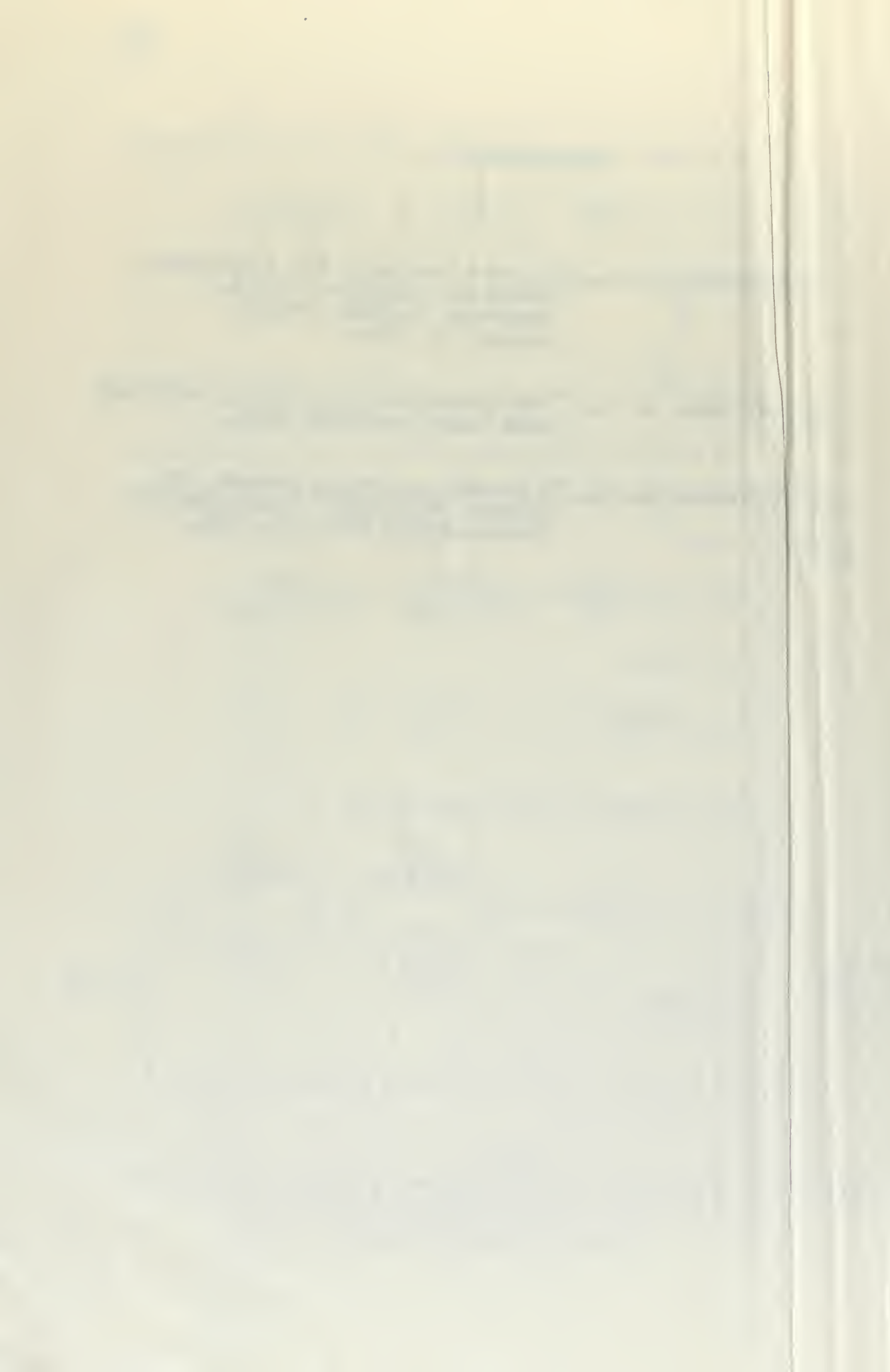
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